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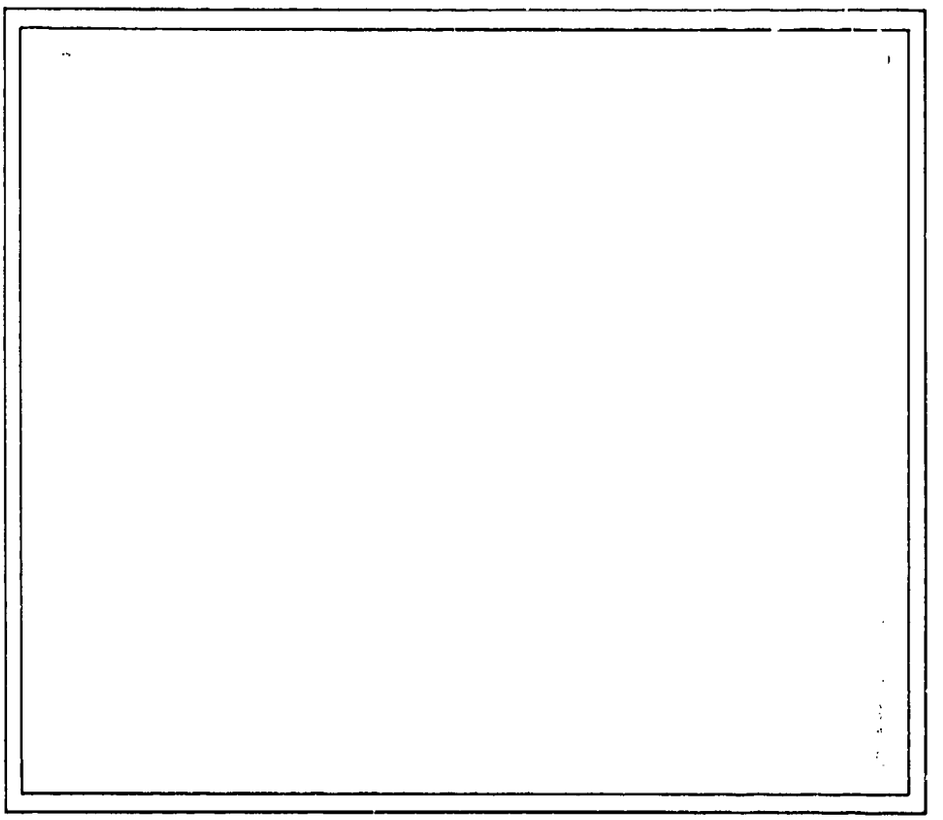

PRINCETON UNIVERSITY

Department of Civil Engineering

STRUCTURES AND MECHANICS

(1) (14)
Technical Rept. No. 46
Civil Engng. Res. Rep. No. 77-SM-10 TR-46

(6) LINE CRACK SUBJECT TO SHEAR

by

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Research Sponsored by the
Office of Naval Research

(15) under
Contract N00014-76-C-0240
Modification No. P00002

(12) 52 p.

(11) July 1977

Approved for Public Release; Distribution Unlimited

401 272 LB

LINE CRACK SUBJECT TO SHEAR[1]

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ABSTRACT

Field equations of nonlocal elasticity are solved to determine the state of stress in the neighborhood of a line crack in an elastic plate subject to a uniform shear at the surface of the crack tip. A fracture criterion based on the maximum shear stress gives the critical value of the applied shear for which the crack becomes unstable. Cohesive stress necessary to break the atomic bonds is calculated for brittle materials.

1. INTRODUCTION

In several previous papers, [1] - [3], we discussed the state of stress near the tip of a sharp line crack in an elastic plate subject to a uniform tension perpendicular to the line of the crack at infinity. The solution of this problem was obtained within the framework of the nonlocal elasticity theory. The resulting solution did not contain the stress singularity present in the classical elasticity solution and therefore a natural fracture criterion based on the usual maximum stress hypothesis could be established. This most interesting outcome could be used to calculate the cohesive stress in various materials.

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[1]The present work was supported by the Office of Naval Research

The present paper deals with the problem of a line crack in an elastic plate where the crack is subject to a uniform shear load. We employ the field equations of nonlocal elasticity theory to formulate and solve this problem. Gratifyingly the resulting solution does not contain the stress singularity at the crack tip and therefore a fracture criterion based on the maximum shear stress hypothesis can be used to obtain the critical value of the applied shear for which the line crack begins to become unstable. If the concept of the surface energy is used it is possible to calculate the cohesive stress holding the atomic bonds together. For steel (with no dislocations) we give an estimate for the cohesive stress. In section 2 we give a resumé of basic equations of the linear nonlocal elasticity theory. In section 3 the boundary value problem is formulated and the general solution is obtained. In section 4 the solution is given for the dual integral equations completing the solution of the problem of line crack subject to a shear load. Calculations for the shear stress are carried out on a computer and results are discussed in section 5.

2. BASIC EQUATIONS OF NONLOCAL ELASTICITY

The basic equations of linear, homogeneous, isotropic, nonlocal elastic solids, with vanishing body and inertia forces, are (cf. [4, 5])

$$t_{k\ell,k} = 0 \tag{2.1}$$

$$t_{k\ell} = \int_V [\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)e_{rr}(\underset{\sim}{x}')\delta_{k\ell}+2\mu'(|\underset{\sim}{x}'-\underset{\sim}{x}|)e_{k\ell}(\underset{\sim}{x}')]dv(\underset{\sim}{x}') \tag{2.2}$$

$$e_{k\ell} = \frac{1}{2}(u_{k,\ell}+u_{\ell,k}) \tag{2.3}$$

where the only difference from classical elasticity is in the stress constitutive equations (2.2) which states that the stress $t_{k\ell}(\underset{\sim}{x})$, at a point $\underset{\sim}{x}$, depends on strains $e_{k\ell}(\underset{\sim}{x}')$, at all points of the body. For homogeneous and isotropic solids the nonlocal elastic moduli $\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ and $\mu'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ are functions of the distance between the variable point $\underset{\sim}{x}'$ and the fixed point $\underset{\sim}{x}$ at which the stress is to be evaluated. The integral in (2.2) is over the volume V of the body enclosed within the surface ∂V.

Throughout this paper we employ cartesian coordinates x_k with the usual convention that a free index takes the values (1, 2, 3) and repeated indices are summed over the range (1, 2, 3). Indices following a comma represent partial differentiation, e.g.

$$u_{k,\ell} \equiv \partial u_k / \partial x_\ell$$

In our previous work [4, 6, 7] we have obtained the forms of $\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ and $\mu'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ for which the dispersion curves of plane waves coincide within the entire Brillouin zone with those obtained in the Born-Von Kármán theory of lattice dynamics. Accordingly

$$(2.4) \qquad (\lambda',\mu') = (\lambda,\mu)\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|) \quad ,$$

$$\alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|) = \begin{cases} \alpha_o(a-|\underset{\sim}{x}'-\underset{\sim}{x}|) & |\underset{\sim}{x}'-\underset{\sim}{x}| \le a, \\ 0 & |\underset{\sim}{x}'-\underset{\sim}{x}| > a \end{cases}$$

where a is the lattice parameter, λ and μ are classical Lamé constants, and α_o is a normalization constant to be determined from

$$(2.5) \qquad \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)dv(\underset{\sim}{x}') = 1$$

While this simple and elegant result is useful in many calculations, it is not the only one that approximates the dispersion curves of lattice dynamics. In fact a very useful one is

$$(2.6) \qquad \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|) = \alpha_o \exp[-(\beta/a)^2(x'_k-x_k)(x'_k-x_k)]$$

where β is a constant. For the two-dimensional case (2.6) has the specific form

$$(2.7) \qquad \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|) = \frac{1}{\pi}(\beta/a)^2 \exp\{-(\beta/a)^2[(x'_1-x_1)^2 + (x'_2-x_2)^2]\}$$

Employing (2.4), in (2.2) we write

$$(2.8) \qquad t_{k\ell} = \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\, \sigma_{k\ell}(\underset{\sim}{x}')dv(\underset{\sim}{x}')$$

where

$$(2.9) \qquad \sigma_{k\ell}(\underset{\sim}{x}') = \lambda e_{rr}(\underset{\sim}{x}')\delta_{k\ell} + 2\mu e_{k\ell}(\underset{\sim}{x}')$$

$$= \lambda u_{r,r}(\underset{\sim}{x}')\delta_{k\ell} + \mu[u_{k,\ell}(\underset{\sim}{x}') + u_{\ell,k}(\underset{\sim}{x}')]$$

is the classical Hooke's law. Substituting (2.8) into (2.1) and using the Green-Gauss theorem we obtain

$$(2.10) \qquad \int_V \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\sigma_{k\ell,k}(\underset{\sim}{x}')dv(\underset{\sim}{x}') - \oint_{\partial V} \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\sigma_{k\ell}(\underset{\sim}{x}')da_k(\underset{\sim}{x}') = 0$$

Here the surface integral may be dropped if the only surface of the body is at infinity.

3. CRACK UNDER SHEAR

Consider a plate in $(x_1 \equiv x,\ x_2 \equiv y)$ - plane weakened by a line crack of length 2ℓ along the x-axis. The plate is subjected to a constant shear stress τ_o along the surfaces of the crack, Fig. 1. For the plane strain problem (2.10) takes the form

$$(3.1) \qquad \text{⨍⨍}_R \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)\sigma_{k\ell,k}(x',y')dx'dy' - \int_{-\ell}^{\ell} \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|)[\underset{\sim}{\sigma}_{2\ell}(\underset{\sim}{x}',0)]dx' = 0$$

where the integral with a slash is over the two-dimensional infinite space excluding the crack line ($|x|<\ell$, $y=0$). A bold-face bracket indicates a jump at the crack line.

When an incision is made in an undeformed body, the body will in general be deformed and stressed because of the long-range interatomic attractions. Thus if we are to treat the problem of a plate with crack, undeformed and unstressed in the natural state, we must consider that after an incision is made the crack is not opened, i.e. the boundary conditions are to be applied to the plate in the natural state.

Under the applied uniform shear load on the unopened surfaces of the crack the displacement field possess the following symmetry regulations

$$(3.2) \qquad u(x,-y) = -u(x,y) \quad , \quad v(x,-y) = v(x,y)$$

Employing this in (2.9) we see that

$$(3.3) \qquad [\underset{\sim}{\sigma}_{2\ell}(x,0)] = 0 \qquad , \qquad |x|>\ell$$

Hence the limits $(-\ell,\ell)$ in the second integral of (3.1) may be replaced by $(-\infty,\infty)$.

The Fourier transform of (3.1) with respect to x' gives

$$(3.4) \qquad \int_{-\infty}^{\infty} \bar{\alpha}(\xi,|y'-y|)[-i\xi\bar{\underset{\sim}{\sigma}}_{1\ell}(\xi,y') + \frac{d}{dy'}\underset{\sim}{\sigma}_{2\ell}(\xi,y')]dy'$$

$$-\bar{\alpha}(\xi,|y|)[\ \bar{\underset{\sim}{\sigma}}_{2\ell}(\xi,0)\ ] = 0$$

where a superposed bar indicates the Fourier transform, e.g.,

$$\bar{f}\ (\xi,y) = (2\pi)^{\frac{1}{2}} \int_{\infty}^{\infty} f(x,y)\exp(i\xi x)d\xi$$

If we take the Fourier transform of (3.4) with respect to y, and solve for the factor of $\bar{\alpha}$ in the integrand of (3.4), upon inversion we obtain

$$-i\xi\bar{\underset{\sim}{\sigma}}_{1\ell}(\xi,y) + \frac{d}{dy}\ \underset{\sim}{\sigma}_{2\ell}(\xi,y) = (2\pi)^{-\frac{1}{2}} \quad [\ \bar{\underset{\sim}{\sigma}}_{2\ell}(x,0)\ ] \int_{-\infty}^{\infty} \exp(-iny)d\eta$$

since the left hand side is defined for all y except y=0. Thus we obtain

$$(3.5) \qquad -i\xi\bar{\underset{\sim}{\sigma}}_{1\ell} + \frac{d\bar{\underset{\sim}{\sigma}}_{2\ell}}{dy} = 0 \qquad , \qquad \ell = 1,2$$

Substituting this into (3.4) we have

$$(3.6) \qquad [\, \sigma_{2\ell}(x,0) \,] = 0 \qquad , \qquad \ell = 1, 2$$

Hence we have shown that the general solution of (3.4) is the same as that of the system (3.5) and (3.6).

The jump condition (3.6) for $|x|>\ell$ is satisfied identically. For $|x|<\ell$ we also have $[\sigma_{21}(x,0)] = 0$, on account of (3.2). Since $\sigma_{22}(x,-y) = -\sigma_{22}(x,y)$ we also see that $t_{22}(x,0) = 0$ for $|x|<\ell$. Thus the normal stress condition on the crack surface is satisfied identically. Considering also the continuity requirement of the displacement field satisfying (3.3) we find that the boundary conditions at x=0 are:

$$(3.7) \qquad \begin{aligned} &\sigma_{yy}(x,0) = 0 \quad , \quad t_{yx}(x,0) = \tau_o \quad , \quad |x|<\ell \\ &\sigma_{yy}(x,0) = 0 \quad , \quad u(x,0) = 0 \quad , \quad |x|>\ell \end{aligned}$$

In addition we must have

$$(3.8) \qquad u = v = 0 \qquad , \qquad \text{as} \quad y \to \infty$$

Consequently we must obtain the solution of (3.5) subject to (3.7) and (3.8).[1]

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[1] Even though some authors feel that $v(x,0)=0$, $|x|>\ell$ should also be satisfied for this (so-called Mode II) problem (cf. [8]), the results based on the boundary conditions (3.7) and (3.8) are accepted by workers on the theory of fracture. The physical reasoning indicates that constant shear load if not balanced by an opposing couple, should give a rotation to the whole body so that $v(x,0)=0$ for $|x|>\ell$ appears to be in contradiction with the expected displacement field.

Equations (3.5) are none other than the Fourier transforms of the Navier's equations in two dimensions, namely

$$\mu\bar{u}_{,yy} - (\lambda+2\mu)\xi^2\bar{u} - i\xi(\lambda+\mu)\bar{v}_{,y} = 0 ,$$

$$-i\xi(\lambda+\mu)\bar{u}_{,y} + (\lambda+2\mu)\bar{v}_{,yy} - \xi^2\mu\bar{v} = 0 \tag{3.9}$$

The general solution of this set (for $y>0$), satisfying (3.8) is.

$$u = -(2\pi)^{-1/2} \int_{-\infty}^{\infty} \xi^{-1} \left[|\xi|A(\xi) + \left(|\xi|y - \frac{\lambda+3\mu}{\lambda+\mu}\right)B(\xi)\right]\exp(-|\xi|y - i\xi x)d\xi,$$

$$v = (2\pi)^{-1/2} \int_{-\infty}^{\infty} i[A(\xi) + yB(\xi)]\exp(-|\xi|y - i\xi x)d\xi \tag{3.10}$$

where $A(\xi)$ and $B(\xi)$ are to be determined from the boundary conditons (3.7). Using (2.9) we calculate

$$\bar{\sigma}_{yy}(\xi,y) = 2\mu i[-|\xi|A(\xi) + \left(\frac{\mu}{\lambda+\mu} - |\xi|y\right) B(\xi)]\exp(-|\xi|y) \tag{3.11}$$

According to $(3.7)_1$ and $(3.7)_3$, this must vanish at $y=0$. Hence

$$B(\xi) = \frac{\lambda+\mu}{\mu} |\xi| A(\xi) \tag{3.12}$$

Noting that $A(-\xi) = A(\xi)$, on account of symmetry $u(x,y)$ $u(-x,y)$, the displacement field may be put into the form

$$u(x,y) = \left(\frac{2}{\pi}\right)^{1/2} \frac{\lambda+\mu}{\mu} \int_0^{\infty} A(\xi) \left(\frac{\lambda+2\mu}{\lambda+\mu} - \xi y\right) e^{-\xi y}\cos(\xi x)d\xi,$$

$$v(x,y) = \left(\frac{2}{\pi}\right)^{1/2} \frac{\lambda+\mu}{\mu} \int_0^{\infty} A(\xi)\left(\frac{\mu}{\lambda+\mu} + \xi y\right) e^{-\xi y}\sin(\xi x)d\xi \tag{3.13}$$

For the $\sigma_{k\ell}$, through (2.9) and (3.12) we obtain (y>0)

$$\sigma_{xx}(x,y) = -\sigma_{xx}(x,-y) = -2(2/\pi)^{1/2}(\lambda+\mu)\int_0^\infty A(\xi)(2\xi-\xi^2 y)e^{-\xi y}\sin(\xi x)d\xi,$$

$$\sigma_{yy}(x,y) = -\sigma_{yy}(x,-y) = -2(2/\pi)^{1/2}(\lambda+\mu)\int_0^\infty A(\xi)\xi^2 y e^{-\xi y}\sin(\xi x)d\xi, \tag{3.14}$$

$$\sigma_{yx}(x,y) = \sigma_{yx}(x,-y) = -2(2/\pi)^{1/2}(\lambda+\mu)\int_0^\infty A(\xi)\xi(1-\xi y)e^{-\xi y}\cos(\xi x)d\xi$$

The stress field according to (2.8), is then given by

$$t_{xx}(x,y) = \int_0^\infty dy' \int_{-\infty}^\infty \sigma_{xx}(x',y')\,[\alpha(|x'-x|,|y'-y|) - \alpha(|x'-x|,|y'+y|)]dx'$$

$$t_{yy}(x,y) = \int_0^\infty dy' \int_{-\infty}^\infty \sigma_{yy}(x',y')\,[\alpha(|x'-x|,|y'-y|) - \alpha(|x'-x|,|y'+y|)]dx' \tag{3.15}$$

$$t_{yx}(x,y) = \int_0^\infty dy' \int_{-\infty}^\infty \sigma_{yx}(x',y')\,[\alpha(|x'-x|,|y'-y|) + \alpha(|x'-x|,|y'+y|)]dx'$$

Substituting for α from (2.7), the integrations may be performed with respect to x' and y' by noting the integrals (cf., [9]).

$$I_1 = \int_{-\infty}^\infty \exp(-px'^2)\begin{Bmatrix}\sin\xi(x+x')\\ \cos\xi(x+x')\end{Bmatrix}dx' = (\pi/p)^{1/2}\exp(-\xi^2/4p)\begin{Bmatrix}\sin\xi x\\ \cos\xi x\end{Bmatrix}$$

$$I_2 = \int_0^\infty \exp(-py'^2-\gamma y')dy' = \tfrac{1}{2}(\pi/p)^{1/2}\exp(\gamma^2/4p)[1-\Phi(\gamma/2\sqrt{p})], \tag{3.16}$$

$$I_3 = \int_0^\infty y'\exp(-py'^2-\gamma y')dy' = \frac{1}{2p} - \frac{\gamma}{4p}\left(\frac{\pi}{p}\right)^{1/2}\exp(\gamma^2/4p)[1-\Phi(\gamma/2\sqrt{p})],$$

Hence,

$$t_{xx} = -(2/\pi)^{\frac{1}{2}}(\lambda+\mu) \int_0^\infty A(\xi) \sin(\xi x) \{e^{-\xi y}[2\xi+(\xi^2/2p)(\xi-2py)]$$

$$\cdot[1-\Phi(\frac{\xi-2py}{2\sqrt{p}})]-e^{\xi y}[2\xi+(\xi^2/2p)(\xi+2py)][1-\Phi(\frac{\xi+2py}{2\sqrt{p}})]\}d\xi,$$

$$t_{yy} = (2/\pi)^{\frac{1}{2}}(\lambda+\mu) \int_0^\infty \xi^2 A(\xi)\sin(\xi x) \{e^{-\xi y} \frac{\xi-2py}{2p}[1-\Phi(\frac{\xi-2py}{2\sqrt{p}})]$$

$$-e^{\xi y} \frac{\xi+2py}{2p} [1-\Phi(\frac{\xi+2py}{2\sqrt{p}})]\}d\xi, \tag{3.17}$$

$$t_{yx} = -(2/\pi)^{\frac{1}{2}}(\lambda+\mu) \int_0^\infty A(\xi)\xi\cos(\xi x) \{e^{-\xi y}[1+(\xi/2p)(\xi-2py)]$$

$$\cdot[1-\Phi(\frac{\xi-2py}{2\sqrt{p}})] + e^{\xi y}[1+(\xi/2p)(\xi+2py)][1-\Phi(\frac{\xi+2py}{2\sqrt{p}})]$$

$$-2\xi(\pi p)^{-\frac{1}{2}}\exp[-py^2-(\xi^2/4p)]\}d\xi$$

where $p=(\beta/a)^2$, and $\Phi(z)$ is the error function defined by

$$\Phi(z) = 2\pi^{-\frac{1}{2}} \int_0^z \exp(-t^2)dt$$

The remaining two boundary conditions $(3.7)_2$ and $(3.7)_4$ may now be expressed as

$$\begin{aligned} &\int_0^\infty \zeta^{\frac{1}{2}} C(\zeta)\, K(\zeta,p) \cos(\zeta z)\, d\zeta = -(\pi/2)^{\frac{1}{2}} T_0 \quad , \quad 0<z<1 \\ &\int_0^\infty \zeta^{-\frac{1}{2}} C(\zeta) \cos(\zeta z)\, d\zeta = 0 \quad , \quad z>1 \end{aligned} \tag{3.18}$$

where we have introduced

$$x/\ell \equiv z \quad , \quad \xi\ell \equiv \zeta \quad , \quad A(\xi) \equiv \zeta^{-\frac{1}{2}}C(\zeta),$$

$$(3.19) \qquad K(\zeta,p) \equiv [1+(\zeta^2/2p\ell^2)][1-\Phi(\zeta/2\ell\sqrt{p})]-(\zeta/\ell\sqrt{\pi p})\exp(-\zeta^2/4p\ell^2),$$

$$T_o \equiv \tau_o\ell^2/2(\lambda+\mu) \quad , \quad p \equiv (\beta/a)^2$$

To determine the unknown function $A(\xi)$ we must solve the dual integral equations (3.18) for $C(\zeta)$.

4. THE SOLUTION OF DUAL INTEGRAL EQUATIONS

By introducing

$$\cos(\zeta z) = (\pi\zeta z/2)^{\frac{1}{2}} J_{-\frac{1}{2}}(\zeta z)$$

where $J_\nu(z)$ is the Bessel's function of order ν, we write the system (3.18) in the form

$$(4.1) \qquad \begin{aligned} &\int_0^\infty \zeta C(\zeta)[1+k(\varepsilon\zeta)]J_{-\frac{1}{2}}(\zeta z)d\zeta = -T_o z^{-\frac{1}{2}} \quad , \quad 0<z<1 \\ &\int_0^\infty C(\zeta)J_{-\frac{1}{2}}(\zeta z)d\zeta = 0 \quad , \quad z>1 \end{aligned}$$

The kernel function $k(\varepsilon\zeta)$ is given by

$$(4.2) \qquad k(\varepsilon\zeta) = K(\zeta,p)-1 = 2\varepsilon^2\zeta^2[1-\Phi(\varepsilon\zeta)]-\Phi(\varepsilon\zeta)-2\pi^{-\frac{1}{2}}\varepsilon\zeta\exp(-\varepsilon^2\zeta^2)$$

where

$$(4.3) \qquad \varepsilon \equiv 1/2\ell p^{\frac{1}{2}} = a/2\beta\ell$$

The solution of the dual integral equations (4.1) is not known. However, it is possible to reduce the problem to the solution of a Fredholm equation (see [10] § 4.6).

$$(4.4) \qquad h(x)+\int_0^1 h(u)L(x,u)du = -\tfrac{1}{2}(\pi x)^{\frac{1}{2}}T_0$$

for the function h(x), where

$$(4.5) \qquad L(x,u) = (xu)^{\frac{1}{2}}\int_0^\infty tk(\varepsilon t)J_0(xt)J_0(ut)dt$$

Once (4.4) is solved then $C(\zeta)$ is calculated by

$$(4.6) \qquad C(\zeta) = (2\zeta)^{\frac{1}{2}}\int_0^1 x^{\frac{1}{2}}J_0(\zeta x)h(x)dx$$

We observe that for $\varepsilon=0$ we have $k=0$, and the dual integral equations (4.1) reduce to those obtained in the classical elasticity. For this case from (4.4) we have $h_0(x) = -T_0(\pi x)^{\frac{1}{2}}/2$, and (4.6) gives the classical result:

$$(4.7) \qquad C_0(\zeta) = -(\pi/2)^{\frac{1}{2}}T_0\zeta^{-\frac{1}{2}}J_1(\zeta)$$

or

$$(4.8) \qquad A_0(\xi) = -(\pi/2)^{\frac{1}{2}}T_0J_1(\xi\ell)/\xi\ell$$

The next iteration of the integral equation (4.4) with the use of $h_0(x)$ gives

$$h_1(x) = -(\pi/2)^{\frac{1}{2}} T_0 \zeta^{\frac{1}{2}} [\zeta^{-1} J_1(\zeta) - I(\zeta,\varepsilon)] \tag{4.9}$$

where

$$I(\zeta,\varepsilon) \equiv \int_0^1 x\, J_0(\zeta x) \hat{f}(x,\xi)dx \tag{4.10}$$

$$= \int_0^\infty \frac{k(\varepsilon t)J_1(t)}{\zeta^2 - t^2} [\zeta J_0(t)J_1(\zeta) - tJ_0(\zeta)J_1(t)]dt \tag{4.11}$$

In which $\hat{f}$ is the Hankel transform of $t^{-1}k(\varepsilon t)J_1(t)$, i.e.,

$$\hat{f}(x,\varepsilon) \equiv \int_0^\infty tf(t,\varepsilon)J_1(xt)dt,$$

$$f(t,\varepsilon) \equiv t^{-1}k(\varepsilon t)J_1(t)$$

If we write

$$x/\varepsilon = y, \quad \varepsilon\zeta = \eta$$

in (4.10), we will have

$$I(\zeta,\varepsilon) = \varepsilon^2 \int_0^{1/\varepsilon} J_0(\eta y)\, \hat{f}(\varepsilon y,\varepsilon)dy$$

But since $\varepsilon^2\hat{f}(\varepsilon y,\varepsilon)$ is the Hankel transform of $f(\eta/\varepsilon,\varepsilon)$, we see that

$$\lim_{\varepsilon\to 0} I(\zeta,\varepsilon) = \lim_{\varepsilon\to 0} f(\zeta,\varepsilon) = \lim_{\varepsilon\to 0} [\zeta^{-1}J_1(\zeta)k(\varepsilon\zeta)]$$

Hence (4.10) vanishes as $k(\varepsilon\zeta)$ with $\varepsilon\to 0$, i.e.,

$$(4.12) \qquad C_1(\zeta) = -(\pi/2)^{\frac{1}{2}} T_o \zeta^{-\frac{1}{2}} J_o(\zeta) + O(\zeta)$$

Thus for small ε the difference between $C_o(\zeta)$ and $C_1(\zeta)$ will be of order ε. Since any number of iterations will contribute higher order terms in ε, the solution for $C(\zeta)$ will also be in the form (4.12) as $\varepsilon\to 0$.

We observe that in general β is in the neighborhood of 1, [6]. The ratio of the atomic distance to the crack length $a/2\ell$ is, however, extremely small for even microscopic cracks. Thus ε is very small and $k(\varepsilon\zeta)$ is in general negligible as compared to unity for all values of $\varepsilon\zeta$. As can be seen from Fig. 2, $k(\varepsilon\zeta)$ goes uniformly from 0 to -1 as $\varepsilon\zeta$ varies between 0 and ∞. Thus we expect that the solution of (4.1) for $C(\zeta)$ will be almost the same as the classical solution $C_o(\zeta)$ corresponding to $\varepsilon=0$. Some differences are of course expected for crack lengths close to atomic distances.

Substituting $A_o(\xi)$ for $A(\xi)$ in (3.13) and (3.17), we obtain the displacement and stress fields. It is of interest to calculate the shear stress along the x-axis. This is given by

$$(4.13) \qquad t_{yx}/\tau_o = \int_o^\infty [1+k(\varepsilon\zeta)] J_1(\zeta)\cos(\zeta z)\,d\zeta \quad , \quad z \equiv x/\ell$$

This integral for $0<z<1$ gives $t_{yx}/\tau_o = 1$ since in this interval the integral converges even for $\varepsilon=0$. For $z>1$ the integral again converges for all $\varepsilon>0$ and it is permissible to ignore $k(\varepsilon\zeta)$ as compared to unity. However for $z=1$ this is no longer the case and we cannot ignore $k(\varepsilon\zeta)$ as compared to unity. To see this we write t_{yx} at $z=1$ as:

$$t_{yx}(x,\ell) = t_1 + t_2 + t_3$$

where

$$t_1 = \tau_o \int_o^\infty J_1(\eta)\ [1-\Phi(\varepsilon\eta)]\cos\eta d\eta,$$

$$t_2 = 2\tau_o \int_o^\infty \varepsilon^2\eta^2 J_1(\eta)\ [1-\Phi(\varepsilon\eta)]\cos\eta d\eta,$$

$$t_3 = -2\tau_o\pi^{-\frac{1}{2}} \int_o^\infty \varepsilon\eta J_1(\eta)\exp(-\varepsilon^2\eta^2)\cos\eta d\eta$$

Since $1-\Phi(\varepsilon\eta)\geq 0$ and $\tau_o \geq 0$ it is clear that

$$|t_1| \leq (\tau_o/\varepsilon) \int_o^\infty [1-\Phi(y)]dy = \tau_o/\pi^{\frac{1}{2}}\varepsilon\ ,$$

$$t_2 \leq (2\tau_o/\varepsilon) \int_o^\infty y^2[1-\Phi(y)]dy = 4\tau_o/3\pi^{\frac{1}{2}}\varepsilon\ ,$$

$$|t_3| \leq (2\tau_o/\pi^{\frac{1}{2}}\varepsilon) \int_o^\infty y \exp(-y^2)dy = \tau_o/\pi^{\frac{1}{2}}\varepsilon$$

Hence

$$|t_{yx}(x,\ell)| \leq (\frac{10}{3}\ \tau_o\pi^{-\frac{1}{2}})\ \varepsilon^{-1}$$

Thus we see that $t_{yx}(x,\ell)$ has finite value for $\varepsilon \neq 0$. In the evaluation of the stress field near $z=1+$ therefore we cannot ignore the function $k(\varepsilon\zeta)$ in (4.13).

5. NUMERICAL ANALYSIS AND DISCUSSION

Calculations of the shear stress t_{yx}, given by (4.13) along the crack line, were carried out on computer. The results are plotted for $\varepsilon=1/20$, $1/50$, $1/100$ and $1/200$ in Figures 3 to 6. For a crack length of 20 atomic distance ($\varepsilon=1/20$) the results are not very good. However for a crack length of 100 atomic distances (Fig. 5) we can see that the shear stress boundary condition $t_{yx}(x,0)=\tau_o$ for $|x|<\ell$ is satisfied in a strong approximate sense. The relative error is less than $1\frac{1}{2}$ %. Hence we conclude that the use of the classical $A_o(\xi)$ given by (4.8) gives satisfactory results for crack lengths greater than 100 atomic distances.

The stress concentration occurs at the crack tip, and it is given by

$$t_{yx}(\ell,0)/\tau_o = c/\sqrt{\varepsilon} \quad , \quad \varepsilon \equiv \beta a/2\ell \tag{5.1}$$

where c converges to about -0.30, i.e.

$$c \doteq -0.30 \tag{5.2}$$

We now make the following significant observations:

(i) The maximum shear stress occurs at the crack tip, and it is finite (eq. 5.1).

(ii) The shear stress at the crack tip becomes infinite as the atomic distance $a\to 0$. This is the classical continuum limit of square root singularity.

(iii) When $t_{yx}(\ell,0) = t_c$ (= cohesive shear stress), fracture will occur. In this case

$$(5.3) \qquad \tau_o^2 \ell = G_c$$

where

$$(5.4) \qquad G_c = (\beta a/2c^2) t_c^2$$

Equation (5.3) is non other than the expression of the Griffith criterion for brittle fracture. Note that we have arrived at this criterion via the maximum shear stress hypothesis. The present criterion of fracture not only unifies the fracture mechanics at the macroscopic and microscopic scales, but also employs the natural concept of bond failure in the atomic scale.

(iv) The cohesive stress t_c may be estimated if one employs the Griffith definition of surface energy γ and writes

$$(5.5) \qquad t_c^2 \, a = K_c \, \gamma \, ,$$

where

$$(5.6) \qquad K_c = 8c^2 \mu / \pi \beta (1-\nu)$$

Employing the values of γ and the elastic constants for steel

$$\gamma = 1975 \text{ CGS} \quad , \quad \mu = 6.92 \times 10^{11} \text{ CGS}$$

$$\nu = 0.291 \quad , \quad a = 2.48 \text{ A}^{\circ} \quad , \quad \beta = 1.65$$

we find that[1]

$$t_c = 1.04 \times 10^{11} \text{ CGS} \quad , \quad t_c/\mu = 0.1$$

This result is in the right range and well accepted by metallurgists based on other considerations. For example Kelly [11] gives $t_c \simeq 6.0 \times 10^{10}$ CGS and $t_c/\mu = 0.11$.

Acknowledgement: The author is indebted to Dr. L. Hajdo for carrying out the computations.

---

[1] The attenuation constant $\beta=1.65$, used in these calculations, makes the Fourier transform of α coincide with the dispersion curve of elastic waves in Born-Von Kárman model of lattice dynamics (cf. [6]).

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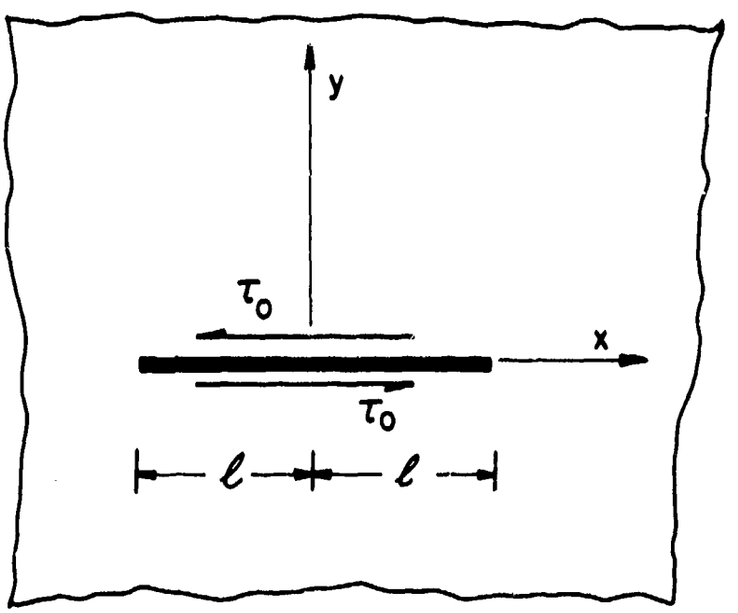
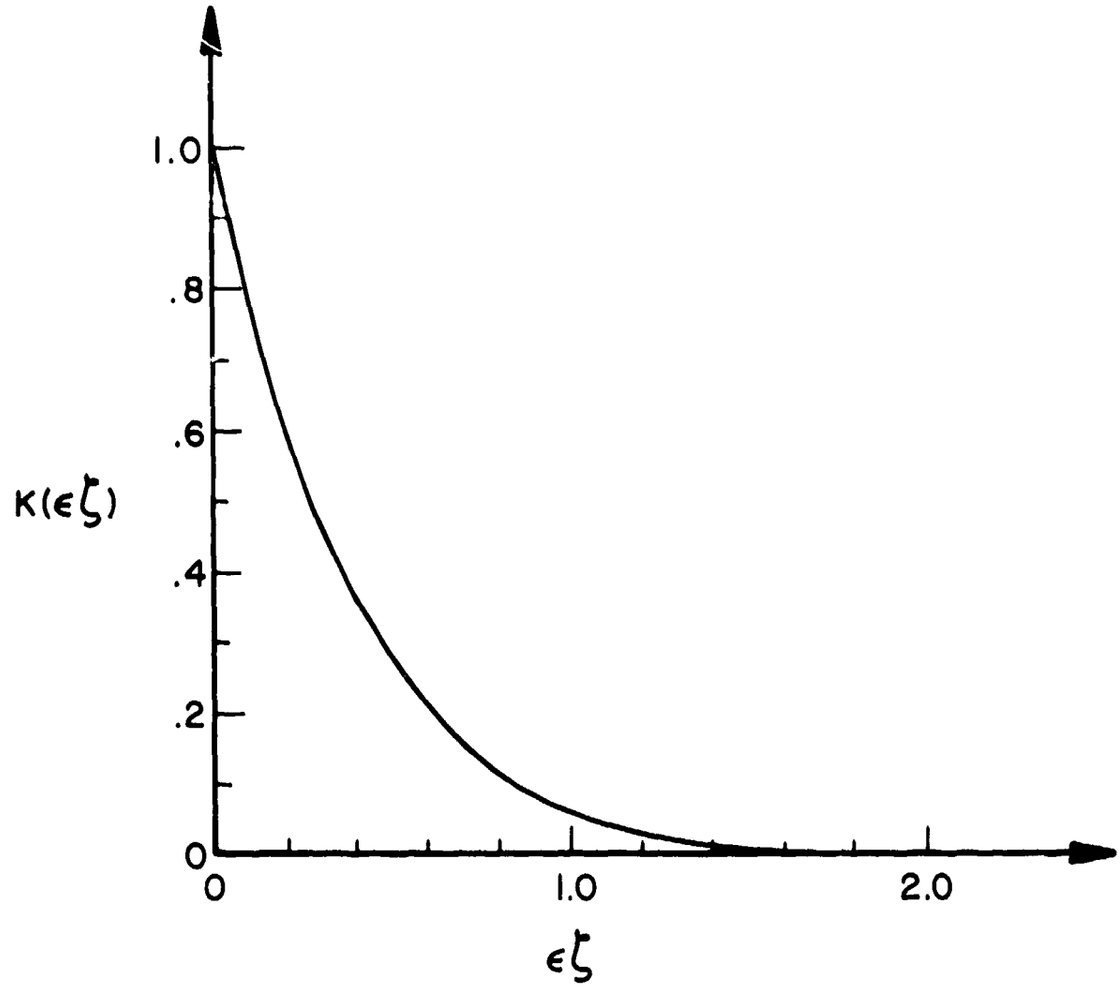


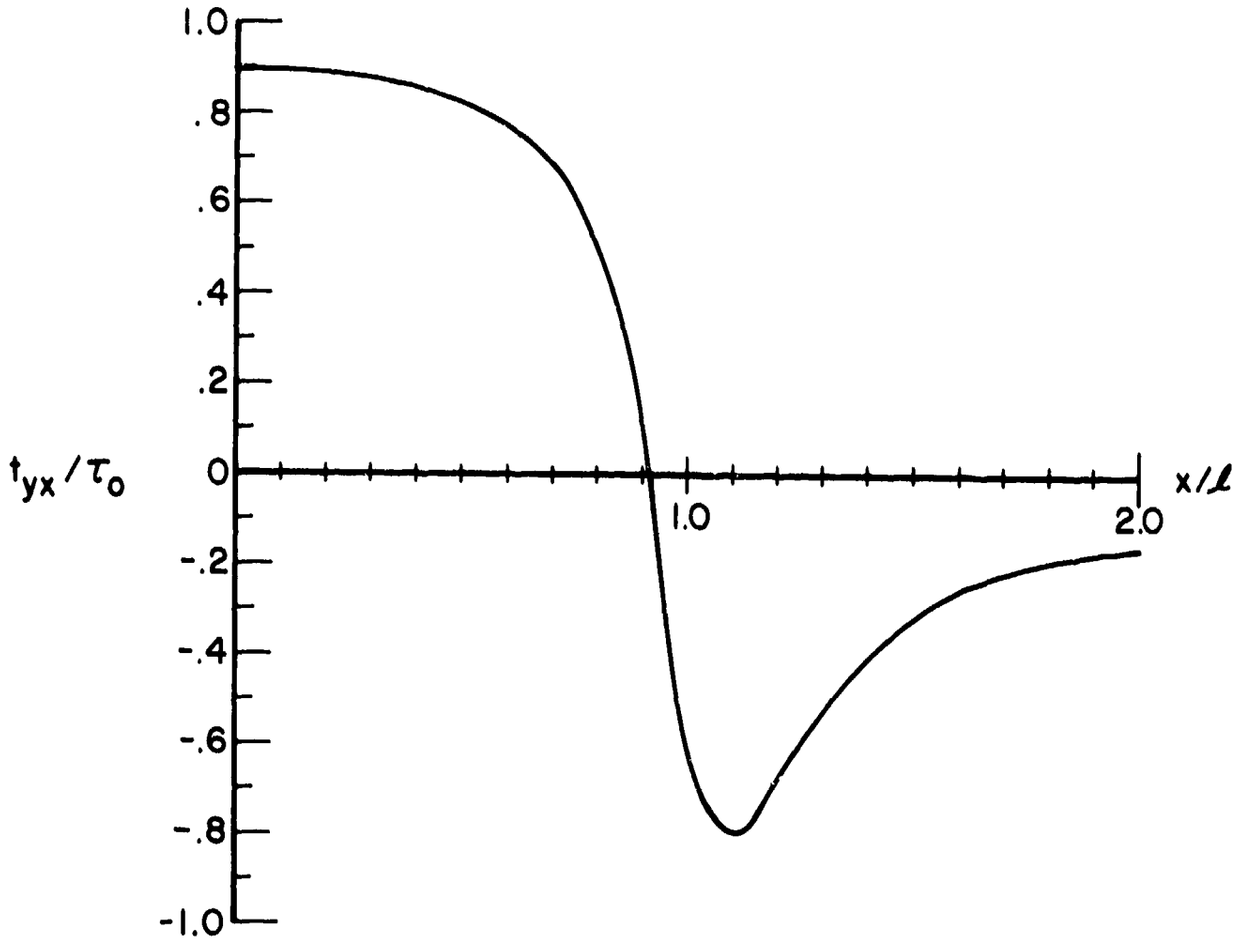
LINE CRACK UNDER SHEAR

FIGURE 1



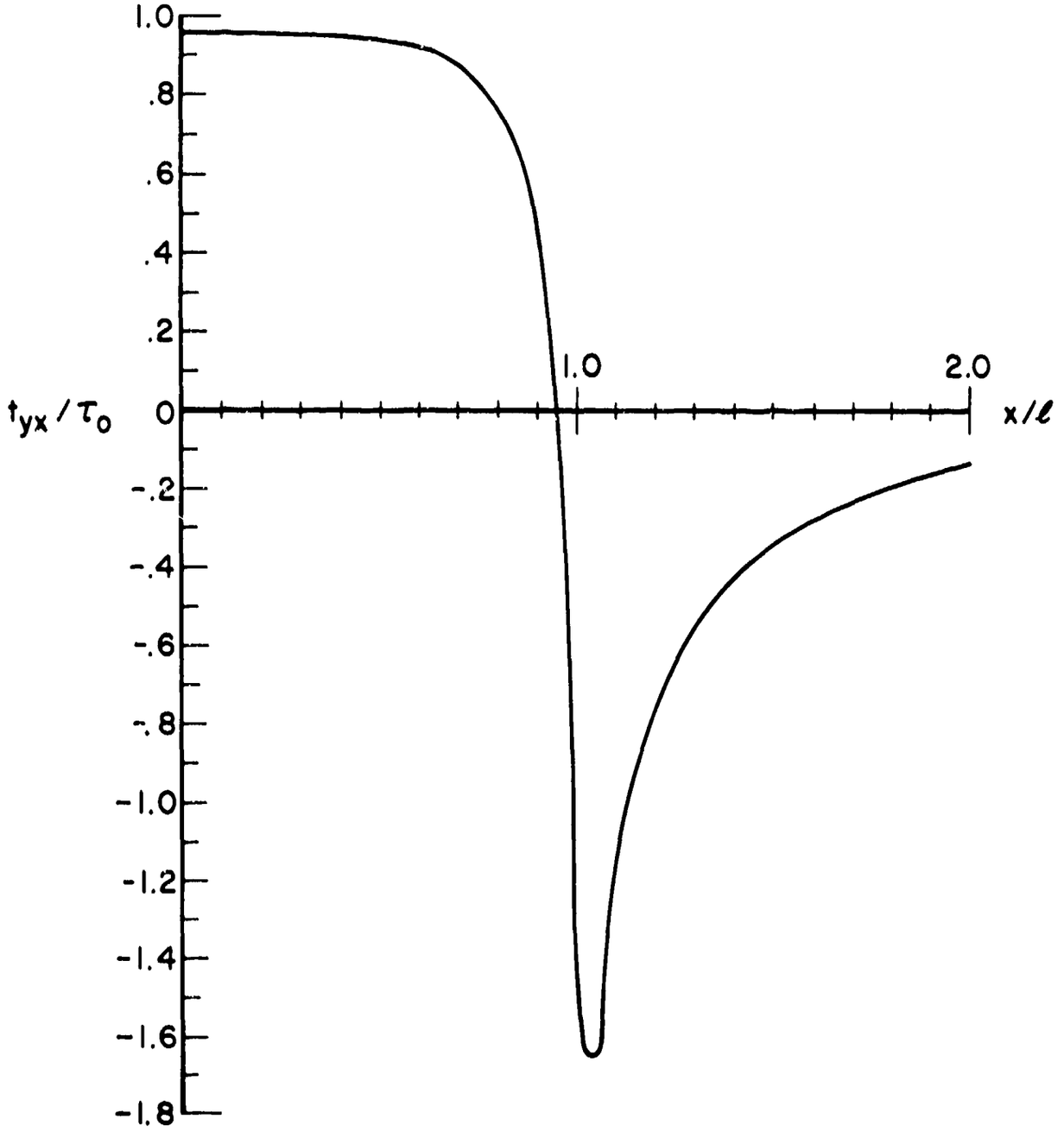
THE BEHAVIOR OF $K(\epsilon\zeta)$ VS $\epsilon\zeta$

FIGURE 2



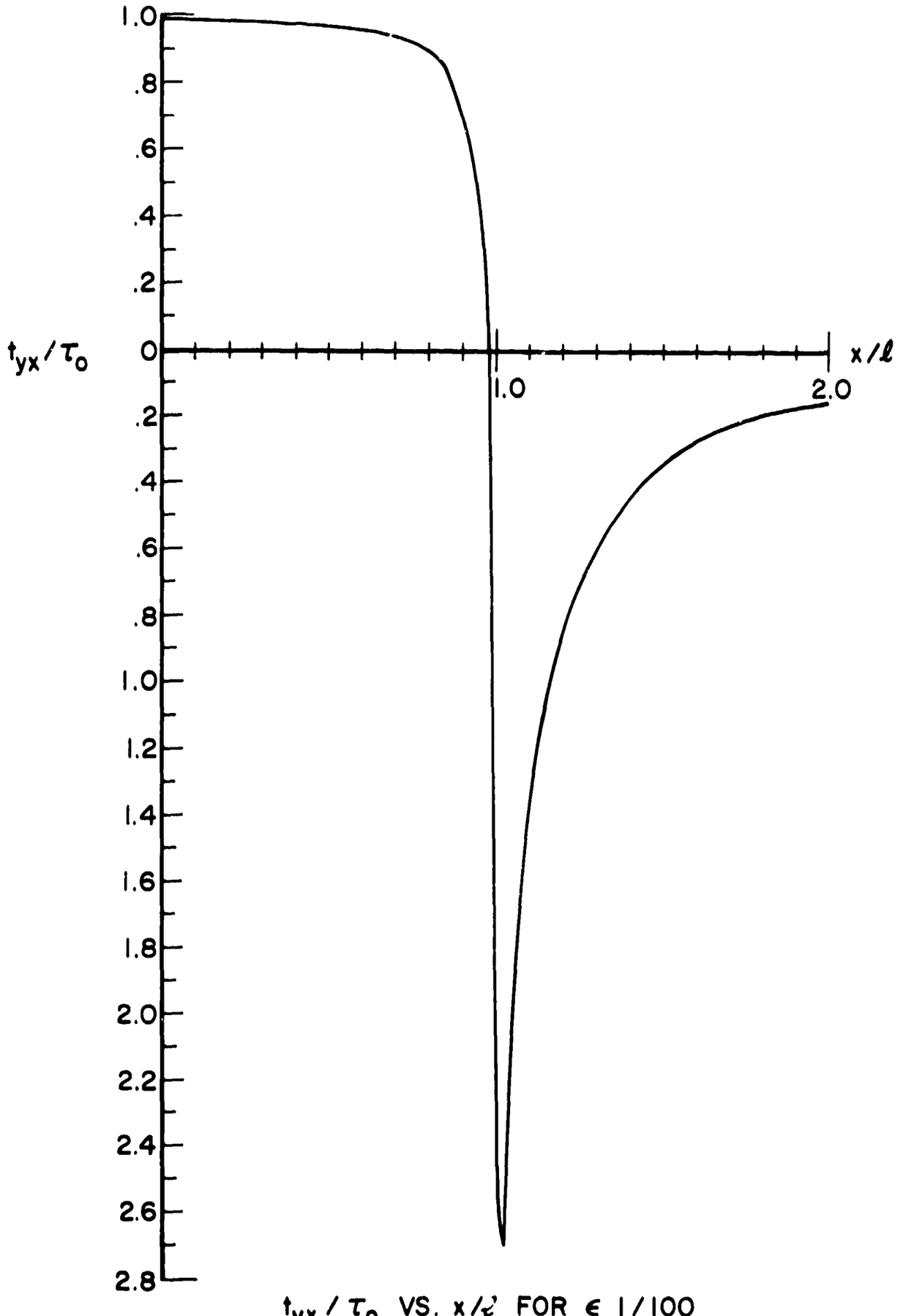
t_{yx}/τ_0 VS. x/ℓ FOR $\epsilon = 1/20$

FIGURE 3



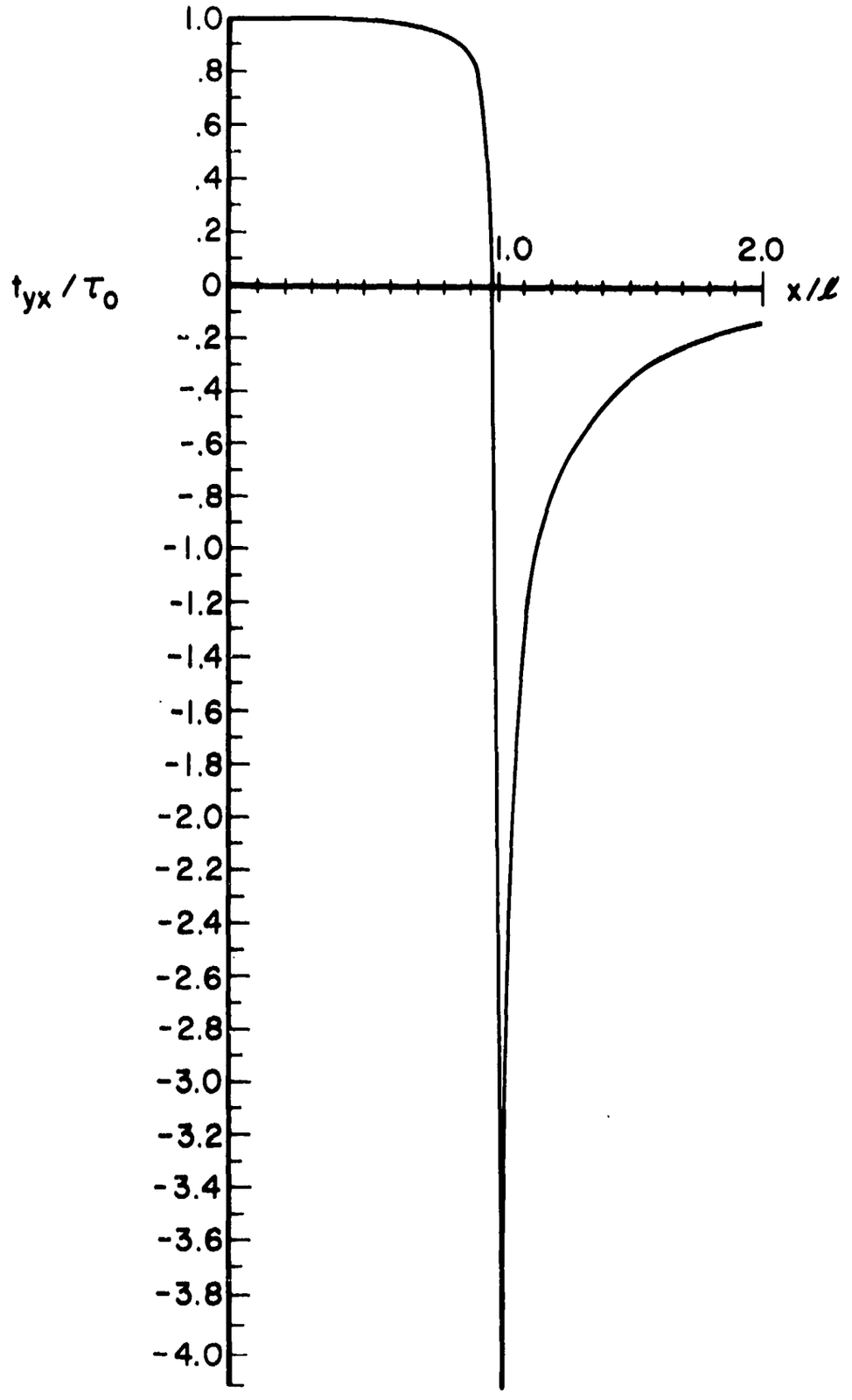
t_{yx}/τ_0 VS. x/ℓ FOR $\epsilon = 1/50$

FIGURE 4



t_{yx}/τ_0 VS. x/ℓ FOR ϵ 1/100

FIGURE 5



t_{yx}/τ_0 VS. x/ℓ FOR $\epsilon = 1/200$

FIGURE 6

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3. RECIPIENT'S CATALOG NUMBER	
4 TITLE (and Subtitle): Line Crack Subject to Shear	5. TYPE OF REPORT & PERIOD COVERED: Technical Report
6. PERFORMING ORG. REPORT NUMBER	
7 AUTHOR(s): A. Cemal Eringen	8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0240
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Princeton University, Princton, New Jersey 08540	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: P00002
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research (Code 471), Arlington, VA 22217	12. REPORT DATE: July 1977
	13. NUMBER OF PAGES: 20
14 MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16 DISTRIBUTION STATEMENT (of this Report)

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17 DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

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19 KEY WORDS (Continue on reverse side if necessary and identify by block number)

line crack
crack under shear
nonlocal elasticity
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Field equations of nonlocal elasticity are solved to determine the state of stress in the neighborhood of a line crack in an elastic plate subject to a uniform shear at the surface of the crack tip. A fracture criterion based on the maximum shear stress gives the critical value of the applied shear for which the crack becomes unstable. Cohesive stress necessary to break the atomic bonds is calculated for brittle materials.

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